The Origin of Matter in the Universe: Reheating after Inflation

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In the inflationary scenario all the matter constituting the universe was created from the process of reheating after inflation. Recent development of the theory of reheating is briefly reviewed. The list of topics includes elementary (perturbative) theory of reheating; quantum field theory in a time-varying background; parametric resonance and explosive particle creation; non-thermal phase transitions from reheating; baryogenesis from reheating; residual oscillations of the scalar field, and other cosmological applications.

1. Introduction

As a graduate student, I studied particle creation from strong gravitational and electric fields. I was impressed by a paper which Ya. Zel'dovich (1972) devoted to J. Wheeler's 60th birthday volume. In particular, Zel'dovich quoted there the "Novikov paradox": let an electron-positron pair be created from the strong electric field. The total momentum of the pair must be equal to zero, but the total energy is not zero. Query: how can this result be the Lorentz-invariant? Answer: the result indeed is the Lorentz-invariant, if the space-time interval between electron and positron is space-like. Hence, at the classical level, the creation of the pair is a causally independent event. The "Novikov paradox" is a fair example of how Igor treats complex physical problems such as quantum creations of particles. Therefore taking occasion to contribute to Igor Novikov's 60th birthday volume, I would like to consider another, one of the most spectacular, applications of the quantum theory of particle creation that emerges together with the Inflationary Scenario. Indeed, almost all matter constituting the Universe at the subsequent radiation-dominated stage was created from the reheating after inflation. 200z The term "reheating" here is an anachronism after the first inflationary models in which the Universe was hot before inflation and was reheated again after inflation. In modern versions of inflationary cosmology the pre-inflationary hot stage is no longer necessary (Linde 1990). It is assumed that the Universe initially expands quasi-exponentially in a vacuum-like state with a vanishing entropy and particle number density. At the stage of inflation, all energy was concentrated in a classical slowly moving inflaton field ϕ. Soon after the end of inflation, when an observable universe was the size of a dime, the inflaton field began to oscillate near the minimum of its effective potential $V(\phi)$. An almost homogeneous inflaton field $\phi(t)$ coherently oscillated with a very large amplitude of the order of the Planck mass $\phi \sim M_p$. The interaction of the inflaton field with other elementary particles led to creation of many ultra-relativistic particles from the classical inflaton oscillations. Gradually, the inflaton field decayed and transferred all its energy non-adiabatically to the created particles. They interacted with each other and came to a state of thermal equilibrium at some temperature T_r, which was called the reheating temperature. An enormous total entropy of the observable universe (or the total number of particles inside the horizon), $S \sim 10^{88}$, was thus produced from the reheating after inflation.

The reheating is an intermediate stage between the inflation and the radiation dominated Friedmann expansion. Therefore the reheating is associated with a number of issues of the Big Bang scenario: the large entropy problem, the baryogenesis problem,

1

the problem of relic monopoles, the problem of topological defects, the primordial black holes problem, etc.

The elementary theory of reheating based on the perturbation theory was developed right after the first models of inflation were suggested. Various aspects of the theory of reheating were further elaborated by many authors. The elementary theory of reheating found its place into the textbooks on the inflation (Linde 1990; Kolb & Turner 1990). Still, the general scenario of reheating in inflationary cosmology was absent, and a number of important problems remained unresolved. In particular, reheating in chaotic inflation theory, which covers many popular models of inflation, remained almost unexplored. Recently, the new effects in particle creation, arising beyond of the perturbation theory, were found, together with further constraints on the elementary theory itself. These effects significantly alter the reheating scenarios. In this contribution we briefly review the theory of the reheating after inflation. We will discuss the different approaches to the theory of reheating: from a quantitative, oversimplified point of view, and within rigorous qualitative formalism. This presentation is based on the ongoing collaboration with Andrei Linde and Alexei Starobinsky.

2. Evolution of the Inflaton Field

We consider a simple chaotic inflation. Among all the fields $(\phi, \chi, \psi, A_i, h_{ik}, ...)$ which are present at these energy scales, the main contribution to gravity comes from the condensate of the scalar field ϕ, or from other condensates which effectively similar to that. In the fundamental Lagrangian $\mathcal{L}(\phi, \chi, \psi, A_i, h_{ik}, ...)$ for large energy density, we can retain for the cosmological applications only gravity and the dominant scalar field

$$\mathcal{L}(R, \phi) = -\frac{M_p^2}{16\pi}R + \frac{1}{2}\phi_i\phi^i - V(\phi). \tag{2.1}$$

The evolution of the FRW universe is described by the Einstein equation

$$H^2 = \frac{8\pi}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) , \tag{2.2}$$

where $H = \dot{a}/a$. The Klein-Gordon equation for $\phi(t)$ is

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 . \tag{2.3}$$

For a sufficiently large initial values of $\phi > M_p$, a "drag" term $3H\dot{\phi}$ in (2.3) dominates over $\ddot{\phi}$; the potential term in (2.2) dominates over the kinetic term. This is the inflationary stage, where the universe expands (quasi-)exponentially, $a(t) = a_0 \exp\left(\int dt H(t)\right)$. However, with a decrease of the field ϕ the "drag" term becomes less and less important, and inflation terminates at $\phi \lesssim M_p/2$. After a short stage of the fast rolling down, the inflaton field rapidly oscillates around the minimum of $V(\phi)$ with the initial amplitude $\phi_0 \sim 0.1 M_p$. Although this value is below the magnitude needed for inflation, it is still a very large figure.

The character of the classical oscillations of the homogeneous scalar field depends on the shape of its potential $V(\phi)$ around the minimum. We will consider two models: the quadratic potential $V(\phi) = \frac{1}{2}m_\phi\phi^2$ and the potential $V(\phi) = \frac{1}{4}\lambda\phi^4$.

For the quadratic potential the solutions of eqs. (2.2) and (2.3) at the stage of oscillations are

$$\phi(t) \approx \phi_0(t) \cdot \sin\left(m_\phi t\right) , \quad \phi_0(t) = \frac{M_p}{\sqrt{3\pi}} \cdot \frac{1}{m_\phi t} . \tag{2.4}$$

The scalar factor averaged over several oscillations is $a(t) \approx a_0 t^{2/3}$. Oscillations of ϕ in

this theory are sinusoidal, with the the amplitude $\phi_0(t) \sim 0.1 M_p a^{-3/2}$ decreasing as the universe expands. The energy of the field ϕ decreases in the same way as the density of nonrelativistic particles of mass m_ϕ: $\epsilon_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2 \sim a^{-3}$. Hence the coherent oscillations of the homogeneous scalar field correspond to the the matter dominated effective equation of states with vanishing pressure.

For the theory with the potential $V(\phi) = \frac{1}{4}\lambda\phi^4$ it is more convenient to express the solutions of eqs. (2.2) and (2.3) via the conformal time $\eta = \int \frac{dt}{a(t)}$:

$$\phi(\eta) \approx \phi_0(\eta) \cdot cn\left(\frac{\omega}{c}\eta, \frac{1}{\sqrt{2}}\right) , \quad \phi_0(\eta) = \sqrt{\frac{3}{2\pi}}M_p \cdot \frac{c}{\omega\eta} . \tag{2.5}$$

The oscillations in this theory are not sinusoidal, but given by elliptic function $cn(\frac{\omega}{c}\eta, \frac{1}{\sqrt{2}})$, with the period of oscillations $\frac{2\pi}{\omega}$, where a numerical constant $c \approx 0.85$ and the effective frequency of oscillations $\omega = c\sqrt{\lambda}a\phi_0 \sim 0.1\sqrt{\lambda}M_p$. With a good accuracy one can write $\phi(\eta) \approx \phi_0(\eta)\sin(c\sqrt{\lambda}a\phi_0\eta)$. The scalar factor is $a(t) \propto \eta(t) \propto \sqrt{t}$. The amplitude of oscillations $\phi_0(t) \sim 0.1 M_p a^{-1}$ decreasing as the universe expands. The energy of the field ϕ decreases in the same way as the density of relativistic particles: $\epsilon_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{4}\lambda\phi^4 \sim a^{-4}$. The effective equation of state corresponds to the radiation dominated equation of states with pressure $p_\phi \approx \frac{1}{3}\epsilon_\phi$.

3. Elementary Theory of Reheating

The amplitude of the oscillations gradually decreases not only because of expansion of the universe, but also because of the energy transfer to particles created by the oscillating field. To describe the decay of the inflaton oscillations, we shall remember again the rest of the fundamental Lagrangian $\mathcal{L}(\phi, \chi, \psi, A_i, h_{ik}, ...)$ which includes the other fields and their interaction with the inflaton field. The inflaton field ϕ after inflation may decay into bosons χ and fermions ψ due to the interaction terms $-\frac{1}{2}g^2\phi^2\chi^2$ and $-h\bar{\psi}\psi\phi$, or into its own Bose quanta $\delta\phi$ due to the self-interaction $\lambda\phi^2\delta\phi^2$, or due to the gravitational interaction (like in the Starobinsky inflationary model). Here λ, g and h are small coupling constants. In case of spontaneous symmetry breaking, the term $-\frac{1}{2}g^2\phi^2\chi^2$ gives rise to the term $-g^2\sigma\phi\chi^2$. We will assume for simplicity that the bare masses of the fields χ and ψ are very small, so that one can write $m_\chi(\phi) \approx g\phi$, $m_\psi(\phi) \approx |h\phi|$.

An elementary theory of reheating based on the perturbation theory was developed by Dolgov & Linde (1982) and by Abbot, Fahri & Wise (1982) for the new inflationary scenario. At the same time the theory of reheating was constructed in the Starobinsky (1982) model. Let us briefly recall the elementary theory of reheating. We consider for simplicity the classical field $\phi(t)$ with the mass m_ϕ oscillating near the minimum of the quadratic potential. A homogeneous scalar field oscillating with frequency $\omega = m_\phi$ can be interpreted as a collection of a number of ϕ-particles with zero momenta. The coherent wave of particles at rest with energy ϵ_ϕ corresponds to particle density $n_\phi = \epsilon_\phi/m_\phi$. Another way around, n_ϕ oscillators of the same frequency m_ϕ, oscillating coherently with the same phase, can be viewed as a single homogeneous wave $\phi(t)$. Based on that interpretation, the effects related to the particle production can be incorporated into the equation of motion (2.3) for the inflaton field by means of the polarization operator (Linde 1990):

$$\ddot{\phi} + 3H\dot{\phi} + \left(m_\phi^2 + \Pi(\omega)\right)\phi = 0 . \tag{3.6}$$

Here $\Pi(\omega)$ is the flat space polarization operator for the field ϕ with the four-momentum $k_i = (\omega, 0, 0, 0)$, $\omega = m_\phi$. The real part of $\Pi(\omega)$ gives only a small correction to m_ϕ^2, but when $\omega \geq min(2m_\chi, 2m_\psi)$, the polarization operator $\Pi(\omega)$ acquires an imaginary

part $\operatorname{Im}\Pi(\omega)$. We will assume that $m_\phi^2 \gg H^2, m_\phi^2 \gg \operatorname{Im}\Pi$. The first condition is automatically satisfied after the end of inflation; the second usually is also true. The solution of (3.6) that generalizes the solution (2.4) and describes damped oscillations of the inflaton field is

$$\phi \approx \frac{M_p}{\sqrt{3\pi}m_\phi t} \exp\left(-\frac{1}{2}\Gamma t\right) \sin\left(m_\phi t\right) , \qquad (3.7)$$

where Γ is the total decay rate of ϕ-particles. Here we used a relation $\operatorname{Im}\Pi = m_\phi \cdot \Gamma$, which follows from unitarity. Thus, eq. (3.7) implies that the amplitude of oscillations of the field ϕ decreases as $\phi_0(t) \approx 0.1 M_p a^{-3/2} \exp\left(-\frac{1}{2}\Gamma t\right)$ due to particle production which occurs during the decay of the inflaton field, as well as due to the expansion of the universe.

For a phenomenological description of this effect one can just add an extra friction term $\Gamma\dot{\phi}$ to the classical equation of motion of the field ϕ, instead of adding the polarization operator (Kolb & Turner 1990):

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + m_\phi^2\phi = 0 . \qquad (3.8)$$

At the stage of oscillations the solution of this equation is similar to (3.7).

One should note, that this equation, just as equation (3.7), is valid only under the conditions $m_\phi \gg H$, $m_\phi \gg \Gamma$, and only at the stage of rapid oscillations of the field ϕ near the minimum of $V(\phi)$. This equation cannot be used to investigate of the stage of slow and fast rolling of the field ϕ during inflation.

Suppose that Γ^{-1} is much less than the typical expansion time of the Universe H^{-1}, the energy density of the inflaton field decreases exponentially within the time Γ^{-1}: $\epsilon_\phi \approx \frac{1}{2}m_\phi^2\phi_0^2 \cdot \exp(-\Gamma t)$. This is exactly the result one would expect on the basis of the interpretation of the oscillating field ϕ as a coherent wave consisting of decaying ϕ-particles.

The rate of decrease of the energy of oscillations coincides with the decay rate of ϕ-particles which is well-known from the perturbation theory. The rates of the processes $\phi \to \chi\chi$ and $\phi \to \psi\psi$ (for $m_\phi \gg 2m_\chi, 2m_\psi$) plus gravitational decay are given by

$$\Gamma_{\phi\to\chi\chi} = \frac{g^4\sigma^2}{8\pi m_\phi} , \quad \Gamma_{\phi\to\psi\psi} = \frac{h^2 m_\phi}{8\pi} , \quad \Gamma_g = \frac{m_\phi^3}{8\pi M_p^2} . \qquad (3.9)$$

Reheating completes at the moment t_r when the rate of expansion of the universe given by the Hubble constant $H = \sqrt{\frac{8\pi\rho}{3M_p^2}} \sim t_r^{-1}$ becomes equal to the total decay rate Γ. The energy density of the universe at the moment $t_r = \Gamma^{-1}$ is

$$\epsilon(t_r) \simeq \frac{3\Gamma^2 M_p^2}{32\pi} . \qquad (3.10)$$

If the particles produced by the decaying inflaton field interact with each other strongly enough, then the thermodynamic equilibrium sets in quickly after the decay of the inflaton field, and the matter acquires a temperature T_r. The energy density of the Universe dominated by the ultrarelativistic particles in a state of thermal equilibrium is

$$\epsilon(T_r) \simeq \frac{\pi^2 g_*}{30} T_r^4 , \qquad (3.11)$$

where a numerical factor $g_*(T_r) \sim 10^2 - 10^3$ depends on the number of ultra-relativistic degrees of freedom. Comparing (3.10) and (3.11) we get the following result for the reheating temperature:

$$T_r \simeq 0.1\sqrt{\Gamma M_p} . \qquad (3.12)$$

Note that T_r does not depend on the initial value of the field ϕ, it is completely determined by the parameters of the underlying elementary particle theory via the decay rate Γ.

In order to get numerical estimates for the duration of reheating t_r, for the decay rate of the inflaton field and for the reheating temperature T_r, one should know the mass of the inflaton field and the coupling constants. We shall take into account several constraints on these in the framework of the perturbation theory (Kofman, Linde & Starobinsky 1996b). The coupling constants of interaction of the inflaton field with matter cannot be too large, otherwise the radiative corrections alter the shape of the inflaton poten- tial. The necessary condition for the decay of inflaton oscillations is $\omega > 2m_\chi, 2m_\psi$. Parameters of the inflaton potential are restricted from the constraints on amplitude of the cosmological fluctuations. All together it allows us to update a constraint on the elementary theory: the largest possible total decay rate in the perturbation theory is $\Gamma < 10^{-20}M_p$. For instance, for the quadratic inflaton potential, it takes at least 10^{14} oscillations to transfer the energy of inflaton oscillations into the created particles. This is a very strong condition, which makes reheating very slow. From (3.12) one can obtain the general bound on the reheating temperature in the model of slow reheating:

$$T_r < 10^9 \, \text{GeV} \ . \tag{3.13}$$

This is a very small temperature, at which the standard mechanism of baryogenesis in the GUTs cannot work; in this case we definitely need a theory of a low-temperature baryoge- nesis. At such a temperature no cosmologically interesting heavy strings, monopoles and textures can be produced, because the generation of the GUT topological defects require the high temperature phase transitions at $T_{GUT} \sim 10^{16} \, \text{GeV}$. For the massive inflaton field, in the slow reheating scenario the post-inflationary matter dominated equation of state of the inflaton oscillations lasts sufficiently long. It can enhance the gravitational instability of density fluctuations, which leads to the formation of the primordial black holes for some specific spectra of the initial density fluctuations.

4. Quantum Field Theory in a Time-dependent Background

Surprisingly, it was found recently that typically the transition between inflation and the hot Big Bang universe may be very different from what the elementary theory pre- dicts (Kofman, Linde & Starobinsky 1994; Shtanov, Traschen & Brandenberger 1995; Boyanovsky *et al* 1995a; Yoshimura 1995). The elementary theory of reheating might be still applicable if the inflaton field can decay into fermions only, with a small coupling constant $h^2 \ll m_\phi/M_p$. This theory also can provide a qualitatively correct description of particle decay at the last stages of reheating. However, the perturbation theory is inapplicable to the description of the first stages of reheating, which makes the whole process quite different. In what follows we will primarily consider the theory of the first stages of reheating. We will begin with the theory of a massive scalar field ϕ decaying into particles χ, then we consider the theory $\frac{\lambda}{4}\phi^4$.

For simplicity, we consider here the interaction $-\frac{1}{2}g^2 F(\phi)\chi^2$ between the *classical* inflaton field ϕ and the *quantum* scalar field χ with the Lagrangian:

$$\mathcal{L}(\chi) = \frac{1}{2}\big(\chi_{,k}\chi^{,k} - m_\chi^2\chi^2 + \xi R\chi^2 - g^2 F(\phi)\chi^2\big) \ . \tag{4.14}$$

Here ξ is a coupling constant of interaction with the space-time curvature R. We also will distinguish two cases, the four-legs interaction $g^2 F(\phi)\chi^2 = g^2\phi^2\chi^2$, and the three-legs interaction $g^2 F(\phi)\chi^2 = 2g^2\sigma\phi\chi^2$ which arises when symmetry is broken. The Heisenberg

representation of the quantum scalar field $\hat{\chi}$ is

$$\hat{\chi}(t, \vec{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_{\vec{k}} \chi_{\vec{k}}(t) e^{-i\vec{k}\vec{x}} + \hat{a}_{\vec{k}}^+ \chi_{\vec{k}}^*(t) e^{i\vec{k}\vec{x}} \right) , \qquad (4.15)$$

where $\hat{a}_{\vec{k}}$ and $\hat{a}_{\vec{k}}^+$ are the annihilation and creation Bose-operators. For the flat Friedmann background with a scalar factor $a(t)$ the temporal part of the eigenfunction with momentum $\vec{k}$ obeys the equation

$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 - \xi R + g^2 F(\phi) \right) \chi_k = 0 . \qquad (4.16)$$

Let us use the conformal time η and introduce the new function $f_k(\eta) = a(\eta)\chi_k(\eta)$. Then instead of (4.16) we have

$$f_k'' + \Omega_k^2 f_k = 0 , \qquad (4.17)$$

where prime stands for $\frac{d}{d\eta}$, and $\Omega_k^2 = (m_\chi a)^2 + k^2 + g^2 a^2 F(\phi) + a^2(1/6 - \xi)R$. This equation describes the oscillators with a variable frequency $\Omega_k^2(t)$ due to the time-dependence of the background field $\phi(\eta)$ and $a(\eta)$. At $\eta \to -\infty$ we choose the positive-frequency solution $f_k(\eta) \simeq \frac{e^{-ik\eta}}{\sqrt{2k}}$. We expect the quantum effect of the χ-particles creation and vacuum polarization as the inflaton field is varying during the slow- and fast-rolling down and oscillations after inflation. The problem is to find a general solution of the equation (4.17) and regularize the formal expressions for the values of the energy-density $\langle \epsilon_\chi \rangle$ and the vacuum expectation $\langle \chi^2 \rangle$.

We adopt a physically transparent method to treat the eq.(4.17) for an arbitrary time dependence of the classical background field which was originally developed by Zeldovich and Starobinsky (1972) for the problem of the particle creation in varying strong gravitational field. In terms of classical waves of the χ-field, quantum effects occur due to the departure from the initial positive-frequency solution. Therefore one may represent solutions of eq.(4.17) as a product of its solutions in the adiabatic approximation, $\exp(\pm i \int d\eta \, \Omega_k)$, multiplied by some functions $\alpha(\eta)$ and $\beta(\eta)$:

$$f_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\Omega_k}} e^{-i \int \Omega_k d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\Omega_k}} e^{+i \int \Omega_k d\eta} . \qquad (4.18)$$

An additional condition on the functions α and β can be imposed by taking the derivative of the expression (4.18) as if α and β would be time-independent. Then the expression (4.18) is a solution of equation (4.17) if the functions α_k, β_k satisfy the equations

$$\alpha_k' = \frac{\Omega_k'}{2\Omega_k} e^{+2i \int \Omega_k d\eta} \beta_k , \qquad \beta_k' = \frac{\Omega_k'}{2\Omega_k} e^{-2i \int \Omega_k d\eta} \alpha_k . \qquad (4.19)$$

Normalization gives $|\alpha_k|^2 - |\beta_k|^2 = 1$, and the initial conditions at $\eta \to -\infty$ are $\alpha_k = 1$, $\beta_k = 0$. The coefficients $\alpha_k(\eta)$ and $\beta_k(\eta)$ in our case coincide with the coefficients of the Bogoliubov transformation of the creation and annihilation operators, which diagonalizes the Hamiltonian of the $\hat{\chi}$-field at each moment of time η.

The regularized vacuum expectation values for the χ^2, energy and particle number densities in terms of $\alpha_k(\eta)$ and $\beta_k(\eta)$ are given correspondingly by

$$\langle \chi^2 \rangle = \frac{1}{2\pi^2 a^2} \int_0^\infty dk \, k^2 \frac{1}{\Omega_k} \left(|\beta_k|^2 + \text{Re} \left(\alpha_k \beta_k^* e^{-2i \int \Omega_k d\eta} \right) \right)_{reg} , \qquad (4.20)$$

$$\langle \epsilon_\chi \rangle = \frac{1}{2\pi^2 a^4} \left(\int_0^\infty dk \, k^2 \Omega_k \, |\beta_k|^2 \right)_{reg} , \qquad \langle n_\chi \rangle = \frac{1}{2\pi^2 a^4} \int_0^\infty dk \, k^2 |\beta_k|^2 . \qquad (4.21)$$

The formal expressions for the vacuum expectation values should be renormalized. The

WKB expansion of the solution of eqs. (4.19) provides a natural scheme of regularization (Zel'dovich & Starobinsky 1972). Thus, the final result is expressed via the coefficients of the Bogoliubov transformation.

The equation of motion of the inflaton field with the feedback of the quantum effects is

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + \frac{1}{2}g^2 \langle \chi^2 \rangle F_{,\phi} = 0 \; , \tag{4.22}$$

where the vacuum average is given by eq.(4.20). The quantum effects contribute to the effective mass of the inflaton field, e.g., for the massive scalar field $m_{eff}^2 = m_\phi^2 + \frac{1}{2}g^2 \langle \chi^2 \rangle F_{,\phi}$. The interactions with fermions $-h\bar{\psi}\psi\phi$ and the self-interaction $-\lambda\phi^2\delta\phi^2$, where $\delta\phi = \hat{\phi} - \phi$, result in extra terms $h < \bar{\psi}\psi >$ and $3\lambda\phi^2\langle\delta\phi^2\rangle$ in (4.22).

With the self-consistent equations (4.20) and (4.22) based on the rigorous quantum field theory in the time varying background, one can investigate the role of the quantum effects on different stages of the evolution of the inflaton field. There are three main quantum effects in an external classical background: vacuum polarization, particle creation and generation of long-wavelength semi-classical fluctuations. Different effects dominate at different stages of the inflationary scenario. During inflation the long-wavelength fluctuations are generated for the minimally coupled light fields with $\xi = 0$, but this effect is irrelevant to the reheating. The vacuum polarization and particle production could occur at the next stage of the fast rolling down of the inflaton field to the minimum of its potential $V(\phi)$ at the end of inflation. However, these effects are negligible and do not alter the dynamics of the inflaton field. At the stage of oscillations of the inflaton field the only important effect is the creation of particles due to the non-adiabatic change of $\Omega_k(\eta)$. The particle production leads to the generation of $\langle\chi^2\rangle$, $\langle\epsilon_\chi\rangle$ and the damping of the inflaton oscillations.

However, the term $\frac{1}{2}g^2\langle\chi^2\rangle F_{,\phi}$ in the equation (4.22) which describes the back reaction of the quantum effects on the evolution of the inflaton field is not reduced to the phenomenological "friction" term $\Gamma\dot{\phi}$, first suggested by Albrecht *et al* (1982). This oversimplified ansatz is widely used to investigate all the stages of the evolution of the inflaton field. Many authors used this method in order to understand whether reheating may slow down the rolling of the field ϕ and, consequently, to support inflation. Remember that one of the main reasons why inflation is possible is the presence of the friction term $3H\dot{\phi}$ in the equation of motion of the scalar field in an expanding Universe. The presence of a similar term due to reheating could support inflation even in the models where inflation would otherwise be impossible. As we seen, the damping of the scalar field during slow and fast rolling down cannot be described in such a simple way. In our study (Kofman, Linde & Starobinsky 1996b) we did not find any possibility to support inflation by reheating. As an exception, the friction term can correctly describe damping of oscillations in the case where the perturbation theory for interaction is applicable.

It is instructive to return in the framework of the rigorous approach to the perturbation theory. Assuming small $|\beta_k| \ll 1$, from eqs. (4.19) one can obtain an iterative solution:

$$\beta_k \simeq \frac{1}{2} \int_{-\infty}^{\eta} d\eta' \frac{\Omega_k'}{\Omega_k} \exp\left(-2i \int_{-\infty}^{\eta'} d\eta'' \Omega_k(\eta'')\right) \; . \tag{4.23}$$

Using expression for $\Omega_k(\eta)$, we obtain that $\beta_k(\eta)$ is proportional to the integral $g^2 \int_{-\infty}^{\eta} d\eta' \, e^{-2ik\eta'} \, (a^2 F(\phi))'$, where $\phi(t)$ is given by the oscillating solutions (2.4) or (2.5), and $F(\phi)$ depends on the type of the interaction. This integral is evaluated by the method of stationary phase (e.g., Starobinsky 1982). To compare the results with these derived in the previous section, let us focus on the case of massive scalar field decaying via the

three-legs interaction $g^2\sigma\phi\chi^2$. In this case the dominant contribution is given by the integration near η_k, where $a(\eta_k) = \frac{2k}{m_\phi}$. This corresponds to the creation of a pair of χ-particles with momentum $k = \frac{1}{2}a(\eta_k)m_\phi$ from an inflaton with the four-momentum $(m_\phi, 0, 0, 0)$ at the moment η_k.

Substituting this estimation for β_k into (4.21), it is easy to calculate the decay rate of the inflaton field. For the three-legs interactions $g\sigma\phi\chi$ or $h\phi\bar\psi\psi$ it is indeed reduced to the formulas (3.9). For the four-legs interaction $\phi\phi \to \chi\chi$, we have creation of a pair of χ-particles with momentum $k = a(\eta_k)m_\phi$ from a pair of massive inflatons with the four-momentum $(m_\phi, 0, 0, 0)$ at the moment η_k. The decay rate of the massive inflaton field in this case is rapidly decreases with the expansion of the Universe as $\frac{1}{a^4}\frac{d}{dt}(a^4\epsilon_\chi) \propto a^{-6}$. Therefore the complete decay of the massive inflaton field in the theory with no spontaneous symmetry breaking or with no interactions with fermions is impossible! This observation gives rise to an unexpected possibility to consider the residual scalar field oscillations as a dark matter candidate. Indeed, if the inflaton oscillations correspond to the matter dominated equation of state, the inflaton field by itself, or other scalar fields with similar properties can be cold dark matter candidates, even if they strongly interact with each other. However, this possibility requires a fine tuning. More immediate application of our result is that it allows one to rule out the inflationary models which predict too small or too large value of the present radiation density parameter Ω_r.

The fact that the number of particles $|\beta_k(\eta)|^2$ with the momentum k is predominantly generated at the certain moment η_k gives us a proper physical interpretation how the particles are created from the coherent oscillations of the homogeneous inflaton field: a pair of particles is created at the instant of the resonance $k = \omega \cdot a(\eta)/2$ between k and the external frequency ω (which is equal m_ϕ for the massive inflaton). As the universe expands, that particular mode is redshifted from the resonance, and creation of particles with the momentum k is terminated leaving $n_k = |\beta_k(\eta)|^2 \ll 1$. At an arbitrary moment η there is a corresponding momentum k resonating with the external frequency.

5. Parametric Resonance: the Stage of Preheating

A new interpretation that particles are created at the instant of resonance with the inflaton oscillations we have grounded in the last section gives us a hint for the general case when n_k is not assumed to be small.

Let us assume there is no symmetry breaking prior reheating. Therefore we will consider oscillations (2.4) of the massive scalar field ϕ decaying into light particles χ due to the interaction $-\frac{1}{2}g^2\phi^2\chi^2$; and oscillations (2.5) in the theory $\lambda\phi^4$ decaying due to the self-interaction $-\frac{3}{2}\lambda\phi^2\delta\phi^2$. In the first case the equation (4.16) for quantum fluctuations $\chi_k(t)$ can be rewritten in the form

$$\frac{d^2(a^{3/2}\chi_k)}{dt^2} + \left(\frac{k^2}{a^2(t)} + +g^2\phi_0^2\sin^2(m_\phi t) + \Delta\right)(a^{3/2}\chi_k) = 0 , \qquad (5.24)$$

where ϕ_0 stands for the amplitude of oscillations of the field ϕ, defined in the formula (2.4), $\Delta = m_\chi^2 + (9/2 - 3\xi)H^2$. As we shall see, the main contribution to χ-particle production is given by excitations of the field χ with $k/a > m_\phi$, which is much greater than H at the stage of oscillations. Therefore, in the first approximation we may neglect the expansion of the Universe, taking $a(t)$ as a constant and omitting the term Δ in (5.24).

In the case of theory with the $\frac{\lambda}{4}\phi^4$ potential the role of quantum field plays the quantum fluctuations $\delta\phi$. In the equations (4.16) for the quantum fluctuations we have to substitute $\chi_k \to \delta\phi_k$, $g^2 \to 3\lambda$. Since in this theory the amplitude $\phi_0(t) \propto a^{-1}(t)$, it is

convenient to use the conformal time η and equations for fluctuations in the form (4.17). For the oscillations (2.5) an exact equation for quantum fluctuations $\delta\phi$ can be reduced to the Lame equation. For simplicity here we will use an approximate equation

$$\frac{d^2(a\delta\phi_k)}{d\eta^2} + \left[k^2 + 3\lambda a^2\phi_0^2 \sin^2(c\sqrt{\lambda}a\phi_0\eta)\right](a\delta\phi_k) = 0 \; , \qquad (5.25)$$

where $c \approx 0.85$, see (2.5). The crucial observation is that the equations (5.24) and (5.25), which describe the fluctuations in the models, can be reduced to the well-known Mathieu equation:

$$\frac{d^2y_k}{dz^2} + (A(k) - 2q\cos 2z)\, y_k = 0 \; , \qquad (5.26)$$

with the periodic effective frequency $\Omega_k^2 = A(k) - 2q\cos 2z$. For the first model (5.24), $y_k = a^{3/2}\chi_k$, $A(k) = \frac{k^2}{\omega^2 a^2}+2q$, $q = \frac{g^2\Phi^2}{4\omega^2}$, $z = \omega t$, the frequency of the inflaton oscillations $\omega = m_\phi$. For the second model (5.25), $y_k = a \cdot \delta\phi_k$, $A \approx \frac{k^2}{\omega^2} + 2.08$, and $q \approx 1.04$, $z = \omega\eta$, the frequency of inflaton oscillations $\omega = c\sqrt{\lambda}a\phi_0$.

An important property of solutions of the equation (5.26) is the existence of an exponential instability $y_k \propto \exp(\mu_k^{(n)}z)$ within the set of resonance bands of frequencies $\Delta k^{(n)}$ labeled by an integer index n. This instability corresponds to exponential growth of occupation numbers of quantum fluctuations $n_{\vec{k}}(t) \propto \exp(2\mu_k^{(n)}m_\phi t)$ that may be interpreted as particle production. The simplest way to analyse this effect is to study the stability/instability chart of the Mathieu equation (MacLachlan 1961).

We will distinguish different regimes of the resonance which correspond to the different regions of the parameters of the instability chart:

Perturbation theory: $q \ll 1$; $A = l^2$, $l = 1, 2, ...$; $2\pi\mu_k \ll H/\omega \ll 1$; $n_k \ll 1$. If no expansion of the universe, fluctuations y_k are slowly generated at the discrete modes k. The expansion quickly redshifts them from the resonance bands. The net effect is reduced to the creation of $n_k \ll 1$ particles with continuous spectrum, which we considered in the previous section.

From this point of view the perturbative regime for the Starobinsky model was considered by Starobinsky (1982) and for the chaotic inflation by Kofman, Linde & Starobinsky (1996b).

Narrow resonance: $q \ll 1$; $A \simeq l^2$, $l = 1, 2, ...$; $2\pi\mu_k \le H/\omega \ll 1$; $n_k \le O(1)$. The universe expansion redshifts the momentum k/a from the narrow resonance band not too fast, so a nonsmall number of particles is generated at the resonance mode. This is an intermediate regime between the perturbation theory and the regime of the broad parametric resonance where expansion of the universe is a subdominant effect.

The narrow resonance was mentioned by Dolgov & Kirilova (1990) for the new inflationary scenario. The importance of this regime for that model was first recognized by Traschen & Brandenberger (1990), but for several reasons their final results were not quite correct. A detailed theory of particle creation in the narrow resonance regime in an expanding universe was developed for the chaotic scenario by Kofman, Linde & Starobinsky (1994; 1996b), see also Shtanov, Traschen & Brandenberger (1995) and Kaiser (1995). In order to investigate the explosive particle production from reheating, many authors considered the model of the narrow resonance without taking into account expansion of the universe because it made investigation simpler (Yoshimura 1995; Boyanovsky *et al.* 1995a,b). However for some range of parameters many vital features of the reheating may disappear in this approximation. In particular, the effects in the model studied by Son (1996) disappear in an expanding universe.

Broad resonance: $q \simeq O(1)$; $A \ge 2q$; $2\pi\mu_k \sim O(1)$; $n_k \gg 1$. Creation of

particles in the regime of a broad resonance is very different from that in the usually considered previous cases. In particular, it proceeds during a tiny part of each oscillation of the field ϕ when $1 - \cos z \sim q^{-1}$ and the induced effective mass of the fluctuations y_k is less than m_{eff}. As a result, the number of the Bose particles grows exponentially fast. This regime occurs only if parameter q is nonsmall. A typical energy E of a particle produced at this stage is determined by equation $A - 2q \simeq 4\sqrt{q}$. The line $A = 2q$ divides the region of the broad resonance from the tachyonic regime where $A \leq 2q$; $\mu_k \geq \pi^{-1}$. The line $A = 2q$ also corresponds to the momentum $k = 0$. Near the line $A = 2q$ typically $\mu_k \sim 0.175$ in the instability bands, with the maximal value about 0.28.

This regime is the most difficult to study. The investigation can be advanced with the following method: consider eq. (5.26) as the equation for the propagation of the wave $y_k(z)$ backward in time. The evolution of $y_k(z)$ can be estimated for each scattering instance, then the net effect is the product of the successive scattering matrixes (Kofman, Linde & Starobinsky 1996b). The results for particle production based on this method were reported by Kofman, Linde & Starobinsky (1994). This stage also was studied by Boyanovsky *et al.* (1995a) and Fujisaki *et al.* (1995).

We shall focus on the most interesting regime of the broad resonance. For our first model (5.24) the necessary condition for it is $m_\phi \ll gM_p$. A typical energy E of a particle produced at this stage is $E \sim \sqrt{gm_\phi M_p}$. As we will see, the broad resonance (preheating) ends up within the short time $t_{ph} \sim m_\phi^{-1} \ln(m_\phi/g^5 M_p)$. As a results, at the end of this stage the occupation number of created particles $n_k \sim exp(2\mu_k m_\phi t)$ with the energy $k = E$ is extremely large:

$$n_E \sim \frac{1}{g^2} \gg 1 \; . \tag{5.27}$$

For another model (5.25), the parameter $q = 1.04$ corresponds to the broad resonance regime. Looking at the instability chart, we see that the resonance occurs in the second band. The typical energy E of a created particles is $E \sim \sqrt{\lambda}a\phi_0$. In the second band maximal value of the coefficient $\mu_k \approx 0.07$. Note that the rigorous equation Lame for the fluctuations in the $\lambda\phi^4$ theory gives a twice smaller value of $\mu_k \approx 0.036$. As long as the backreaction of created particles is small, expansion of the Universe does not shift fluctuations away from the resonance band, and the number of particles $n_k \sim \exp(\frac{\sqrt{\lambda}\phi_0}{10}t)$. The broad resonance in this model ends up within the time interval $t_{ph} \sim M_\phi^{-1}\lambda^{-1/2}|\ln \lambda|$. At the end of this stage the occupation number is

$$n_E \sim \frac{1}{\lambda} \gg 1 \; . \tag{5.28}$$

In addition to this models, the field ϕ in the theory $\lambda\phi^4$ may decay to χ-particles due to the interaction $-\frac{1}{2}g^2\phi^2\chi^2$. This is the leading process for $g^2 \gg \lambda$. The parametric resonance is broad. The values of the parameter μ_k along the line $A = 2q$ do not change monotonically, but typically for $q \sim g^2/\lambda \gg 1$ they are 3 to 4 times greater than the parameter μ_k for the decay of the field ϕ into its own quanta. Therefore, the resonance in this theory is very efficient.

The stage of broad parametric resonance which leads to the explosive particle creation is lasting typically for a few dozens of oscillations (for several oscillations in the last model). However, this does not mean that the process of reheating has been completed. Instead of created particles in the thermal equilibrium, by the end of this stage one has particles far away of equilibrium but with extremely large mean occupation numbers (5.27), (5.28). It is therefore convenient to divide the whole process of reheating into three different stages. At the first stage, which cannot be described by the elementary

theory of reheating, the classical coherently oscillating inflaton field ϕ decays into bosons due to parametric resonance. In many models the resonance is broad, and the process occurs extremely rapidly (explosively). Because of the Pauli exclusion principle, there is no explosive creation of fermions. To distinguish this stage from the subsequent stages when particles decay or interact, we call it *pre-heating*. The second stage is the evolution of the system after preheating. During this stage the particles produced from the preheating, can decay into other particles and self-interact. The last stage is the stage of thermalization, which can be described by standard methods.

6. Back Reaction of Created Particles

Creation of particles leads to the several effects which can change the dynamics of the system. In particular, it terminates the broad resonance particle production. First, the energy from the homogeneous field $\phi(t)$ is transferred to the created particles. The amplitude of the classical oscillations ϕ_0 is therefore decreasing faster than it would decrease due to the expansion of the universe only. Second, as it follows from (4.22), fluctuations contribute to the effective frequency of the inflaton oscillations: $\omega_{eff}^2 = m_\phi^2 + g^2\langle\chi^2\rangle$ or $\omega_{eff}^2 = c^2\lambda\phi^2 + 3\lambda\langle\,\delta\phi^2\rangle$. Third, fluctuations also can contribute to the effective mass of fluctuations themselves. As a result, the q parameters in the Mathieu equation are changing, in the different models correspondingly as $q = g^2\phi_0^2/4\omega_{eff}^2$ and $q = 3\lambda a^2\phi_0^2/4\omega_{eff}^2$. The $A(k)$ parameters are changing as $A(k) = k^2/\omega_{eff}^2 a^2 + 2q$, and $A(k) = (k^2 + 3\lambda a^2\langle\,\delta\phi^2\rangle)/\omega_{eff}^2 + 2q$. It moves the resonance momenta k out the their initial positions on the instability chart. The parameter q is decreasing, which dynamically shifts the parameters towards the narrower resonance.

As a result, the creation of particles in the broad resonance regime is terminated. A simple criterion of the end of the preheating is the condition that the energy density of produced fluctuations is of the same order as the energy density the scalar field at the end of preheating t_{ph}. For the first model $\langle\chi^2\rangle \sim \phi_0^2$, for the second model $\langle\,\delta\phi^2\rangle \sim \phi_0^2$. From this the timing of the preheating t_{ph} given above was estimated.

After copious particle creation around the resonance mode E slowed down, the inflaton field still continues to transfer its energy into the energy of fluctuations. Son (1996) noticed that self-interaction of the $\delta\phi$ particles in the $\lambda\phi^4$ theory might be important since they have very large occupation number density $n_E \sim 1/\lambda$ at the resonance mode. Indeed, the usual estimation for the relaxation time τ for the diluted gas of the $\delta\phi$ particles due to the self-interaction has limited relevance when the occupation number density of particles is large. The scattering of created particles, however, does not eliminate the preheating regime. The particles created from the resonance, in the momentum space occupy only very narrow shells corresponding to successive resonance bands. Most of the particles after preheating are located in the first shell of the radius E and the width $\sim 0.1E$ (additionally their occupation number n_k is sharply decreases towards the edges of the shell). Two particles from this shell can be scattered inside the shell. The rate of redistribution of particles within the shell is indeed much larger than τ^{-1}, but it does not terminate the resonance effect since particles remain inside the resonance shell. Two particle from the shell can be scattered outside of the resonance shell. However, this process is much slower than the preheating time t_{pr}. There are interesting possibilities that some of the particles from the first resonance shell are scattered into other resonance shells. However, the whole process of rescattering and further creation of particles in the narrow resonance after preheating is rather complicated. Khlebnikov & Tkachev (1996) performed first numerical simulation of the nonlinear evolution of the inflaton

field in $\lambda\phi^4$ theory with rescattering. Based on the result of Polarski & Starobinsky (1996) that the particles created with the large occupation number can be viewed as the superposition of the classical waves with random phases, they modelled preheating and scattering of ϕ particles as the evolution of classical random gaussian field with the spectrum which has a sharp spike with the amplitude (5.28) at the mode E superimposed with the homogeneous background. Numerical simulation confirms the features of preheating derived analytically. They also show the further growth of the fluctuations $\langle\delta\phi^2\rangle$ and reveal its spectrum after the preheating in this toy model. We shall note that in the more general case, when inflaton field can decay into other bosons, say, due to $g^2\phi^2\chi^2$-interaction, the rescattering is less significant. We also do not find evidence of the strong Bose condensations at $k = 0$.

7. Non-Thermal Phase Transitions from After Inflation

As we have already seen, the reheating scenario depends on the type and strength of interactions of the involved fields. In this section we show that the reheating also strongly depends on the structure of the elementary particle theory. So far we considered theories with no symmetry breaking. In the case with symmetry breaking, in the beginning, when the amplitude of oscillations ϕ_0 is much greater than σ, the theory of decay of the inflaton field is the same as in the case considered above. The most important part of pre-heating occurs at this stage. When the amplitude of the oscillations becomes smaller than $m_\phi/\sqrt{\lambda}$ and the field begins oscillating near the minimum of the effective potential at $\phi = \sigma$, particle production due to the narrow parametric resonance typically becomes weak. The main reason for this is related to the backreaction of particles created at the preceding stage of pre-heating on the rate of expansion of the universe and on the shape of the effective potential. However, importance of spontaneous symmetry breaking for the theory of reheating should not be underestimated, since it gives rise to the interaction term $g^2\sigma\phi\chi^2$ or $\lambda\sigma\phi\delta\phi^2$ which is linear in ϕ. Such terms are necessary for a complete decay of the inflaton field in accordance with the perturbation theory.

However, in the theories where preheating is possible one may expect many unusual phenomena. One of the most interesting effects is the possibility of specific non-thermal post-inflationary phase transitions which occur after preheating (Kofman, Linde & Starobinsky 1996a, Tkachev 1996). These phase transitions in certain cases can be much more pronounced that the standard high temperature cosmological phase transitions (Kirzhnits & Linde 1972, Linde 1990). They may lead to copious production of topological defects and even to a secondary stage of inflation after reheating.

Let us first remember the theory of phase transitions in theories with spontaneous symmetry breaking. For simplicity let us consider the theory of scalar fields ϕ the effective potential

$$V(\phi, \chi) = \frac{\lambda}{2}(\phi^2 - \sigma^2)^2 \ . \tag{7.29}$$

$V(\phi)$ has a minimum at $\phi = \sigma$, and a maximum at $\phi = 0$ with the curvature $V_{\phi\phi} = -m^2 = -\lambda\sigma^2$. This effective potential acquires corrections due to quantum (or thermal) fluctuations of the scalar fields $\Delta V = \frac{3}{2}\lambda\langle(\delta\phi^2)\rangle\phi^2$. In the large temperature limit $\langle(\delta\phi)^2\rangle = \frac{T^2}{12}$. The effective mass squared of the field ϕ

$$m_{\phi,eff}^2 = -m^2 + 3\lambda\phi^2 + 3\lambda\langle(\delta\phi)^2\rangle \tag{7.30}$$

becomes positive and symmetry is restored (i.e. $\phi = 0$ becomes the stable equilibrium

point) for $T > T_c$, where $T_c^2 = \frac{4m^2}{\lambda} \gg m^2$. At this temperature the energy density of the gas of ultrarelativistic particles is given by $\rho = g_*(T_c)\frac{\pi^2}{30}T_c^4 = \frac{8\,m^4 N(T_c)\pi^2}{15\,\lambda^2}$.

The theory of cosmological phase transitions is an important part of the theory of the evolution of the universe, and during the last twenty years it was investigated in a very detailed way. However, typically it was assumed that the phase transitions occur in the state of thermal equilibrium. Now we are going to show that similar phase transitions may occur even much more efficiently prior to thermalization, due to the anomalously large expectation values $\langle(\delta\phi)^2\rangle$ produced during the first stage of reheating after inflation.

We will first consider the model (7.29) with the amplitude of spontaneous symmetry breaking $\sigma \ll M_p$. As we seen in the previous sections, during preheating inflaton oscillates transfer most of its energy $\sim \lambda M_p^4$ to its long-wave fluctuations $\langle(\delta\phi)^2\rangle \sim M_p^2$ in the regime of broad parametric resonance.

The crucial observation is the following. If the initial energy density $\sim \lambda M_p^4$ were instantaneously thermalized, the reheating temperature $T_r \sim \lambda^{1/4}M_p$ would be much greater than the typical particle energy after preheating $E_\phi \sim \sqrt{\lambda}M_p$, and the magnitude of fluctuations $\langle(\delta\phi)^2\rangle \sim T_r^2/12 \sim \sqrt{\lambda}M_p^2$ would be much smaller than the magnitude of non-thermalized fluctuations $\langle(\delta\phi)^2\rangle \sim M_p^2$. Thus after the first stage of reheating, the non-thermalized fluctuations of the scalar field ϕ are much greater than the thermalized ones. Thermal fluctuations would lead to symmetry restoration in our model only for $\sigma \lesssim T_r \sim \lambda^{1/4}M_p$. Meanwhile, according to eq. (7.30), the non-thermalized fluctuations $\langle(\delta\phi)^2\rangle \sim M_p^2$ may lead to symmetry restoration in our model even if the symmetry breaking parameter σ is as large as M_p. Thus, the non-thermal symmetry restoration occurs even in those theories where the symmetry restoration due to high temperature effects would be impossible. Later on, $\langle(\delta\phi)^2\rangle \propto a^{-2}$, $E_\phi \propto a^{-1}$ because of the expansion of the universe (as far as $E_\phi \gg m$). This leads to the phase transition with symmetry breaking at the moment $t = t_c \sim \sqrt{\lambda}M_p m^{-2}$ when $m_{\phi,eff} = 0$, $\langle(\delta\phi)^2\rangle = \sigma^2/3$, $E_\chi \sim m$. Note that the homogeneous component $\phi(t)$ at this moment is significantly less than $\sqrt{\langle(\delta\phi)^2\rangle}$ due to its further decay after preheating.

The mechanism of symmetry restoration described above is very general; in particular, it explains a surprising behavior of oscillations of the scalar field found numerically in the $O(N)$-symmetric model of Boyanovsky *et al.* (1995a). It is important that during the interval between preheating and the establishment of thermal equilibrium the universe could experience a series of phase transitions which we did not anticipate before. For example, cosmic strings and textures, which could be an additional source for the formation of the large scale structure of the universe, should have $\sigma \sim 10^{16}$ GeV. To produce them by thermal phase transitions in our model one should have the temperature after reheating greater than 10^{16} GeV, which is extremely hard to obtain (Kofman & Linde 1987). Meanwhile, as we see now, fluctuations produced at preheating may be quite sufficient to restore the symmetry. Then the topological defects are produced in the standard way when the symmetry breaks down again. In other words, production of superheavy topological defects can be easily compatible with inflation.

On the other hand, the topological defect production can be quite dangerous. For example, the model (7.29) of a one-component real scalar field ϕ has a discrete symmetry $\phi \to -\phi$. As a result, after the phase transition induced by fluctuations $\langle(\delta\phi)^2\rangle$ the universe may become filled with domain walls separating phases $\phi = +\sigma$ and $\phi = -\sigma$. This is expected to lead to a cosmological disaster. Investigation shows that in the theory (7.29) with $\sigma \ll 10^{16}$ Gev fluctuations destroy the coherent distribution of the background oscillations of ϕ and divide the universe in an equal number of domains with $\phi = \pm\sigma$, which leads to the domain wall problem. This mean that in consistent

inflationary models of the type of (7.29) one should have either no symmetry breaking or $\sigma \geq 10^{16}$ Gev.

Similar effects of the nonthermal phase transitions occur in the models where the symmetry breaking occurs for fields other than the inflaton field ϕ (Kofman, Linde & Starobinsky 1996a). The simplest model has an effective potential (e.g., Kofman & Linde 1987; Linde 1994):

$$V(\phi, \chi) = \frac{\lambda}{4}\phi^4 + \frac{\alpha}{4}\left(\chi^2 - \frac{M^2}{\alpha}\right)^2 + \frac{1}{2}g^2\phi^2\chi^2 \ . \tag{7.31}$$

We will assume here that $\lambda \ll \alpha < g^2$, so that at large ϕ the curvature of the potential in the χ-direction is much greater than in the ϕ-direction. In this case at large ϕ the field χ rapidly rolls toward $\chi = 0$. An interesting feature of such models is the symmetry restoration for the field χ for $\phi > \phi_c = M/g$, and symmetry breaking when the inflaton field ϕ becomes smaller than ϕ_c. As was emphasized by Kofman & Linde (1987), such phase transitions may lead to formation of topological defects without any need for high-temperature effects.

Now we would like to point out some other specific features of such models. If the phase transition discussed above happens during inflation (i.e. if $\phi_c > M_p$ in our model), then no new phase transitions occur in this model after reheating. However, for $\phi_c \ll M_p$ the situation is much more complicated. First of all, in this case the field ϕ oscillates with the initial amplitude $\sim M_p$ (if $M^4 < \alpha\lambda M_p^4$). This means that each time when the absolute value of the field becomes smaller than ϕ_c, the phase transition with symmetry breaking occurs and topological defects are produced. Then the absolute value of the oscillating field ϕ again becomes greater than ϕ_c, and symmetry restores again. However, this regime does not continue for a too long time. Within several oscillations, quantum fluctuations of the field χ will be generated with the dispersion $\langle(\delta\chi)^2\rangle \sim g^{-1}\sqrt{\lambda}M_p^2$. For $M^2 < g^{-1}\sqrt{\lambda}\alpha M_p^2$, these fluctuations will keep the symmetry restored. The symmetry breaking will be finally completed when $\langle(\delta\chi)^2\rangle$ will become small enough. Riotto and Tkachev (1996) noted that in the model (7.31) in the case of the strong self-interaction of χ particles, $\lambda \ll g^2 \ll \alpha$, the presence of a huge occupation number of the created χ particles $n_E \sim 1/g^2$ would require the consideration beyond the one-loop approximation. It can be shown, however, that in this case the the preheating stops earlier when $n_E \sim 1/\alpha$ (Kofman, Linde & Starobinsky 1996b).

One may imagine even more complicated scenario when oscillations of the scalar field ϕ create large fluctuations of the field χ, which in their turn interact with the scalar fields Φ breaking symmetry in GUTs. Then we would have phase transitions in GUTs induced by the fluctuations of the field χ. Note that in the models considered in this section the field χ does not oscillate near $\chi = 0$ prior to the phase transition, since such oscillations are damped out during the long stage of inflation prior to the phase transition. Thus oscillations of the field χ in the theory (7.31) definitely do not suppress the topological defect production. This means that no longer can the absence of primordial monopoles be considered as an automatic consequence of inflation. To avoid the monopole production one should use the theories where quantum fluctuations produced during preheating are small or decoupled from the GUT sector. This condition imposes additional constraints on realistic inflationary models.

8. Conclusions

We have found that the decay of the inflaton field typically begins with the explosive production of particles during the stage of preheating in the regime of a broad parametric

resonance. During the second stage, the inflaton field decays further in the regime of the narrow resonance, and particles produced at this stage decay into other particles and self-interacting. The third stage is the thermalization. The two last stages require much more time that the stage of preheating. Typically, coupling constants of interaction of the inflaton field with matter are extremely small, whereas coupling constants involved in the decay of other particles can be much greater. As a result, the reheating temperature T_r given by (3.12) will not necessarily be defined by the perturbative decay rate of the inflaton oscillations (3.9). T_r can be much higher than the typical temperature $T_r \lesssim 10^9$ GeV, which could be obtained neglecting the stage of parametric resonance.

A specific feature of the preheating is that bosons produced at this stage are far away from thermal equilibrium and typically have enormously large occupation numbers. This may have very interesting applications to cosmology. There is a new class of phase transitions that may occur at the intermediate stage between the end of inflation and the establishment of thermal equilibrium. These phase transitions take place even in the theories where the scale of spontaneous symmetry breaking is comparable to M_p and where the reheating temperature is very small. Therefore, phase transitions of the new type may have dramatic consequences for inflationary models and the theory of physical processes in the very early universe.

The preheating may help with the baryogenesis problem (Kofman, Linde & Starobinsky 1996a; Yoshimura 1996). In the models of GUT baryogenesis, it was assumed that GUT symmetry was restored by high-temperature effects, since otherwise the density of X, Y, and superheavy Higgs bosons would be very small. This condition is hardly compatible with inflation. It was also required that the products of decay of these particles should stay out of thermal equilibrium, which is a very restrictive condition. In our case, the superheavy particles responsible for baryogenesis can be abundantly produced by parametric resonance, and they as well as the products of their decay will be out of thermal equilibrium until the end of reheating.

Another consequence of the resonance effects is a fast change of the equation of state from a vacuum-like one to the equation of state of relativistic matter. This leads to suppression of primordial black holes formation that could be produced after inflation.

The preheating stage may appear not only in the models where the reheating occurs due to the decay of the homogeneous scalar field. Kolb & Riotto (1996) suggested that preheating may also occur in first order inflation. It would be very interesting to investigate this problem, as well as the many other exciting possibilities which the new theory of reheating may offer us.

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